H

National
Qualifications
SPECIMEN ONLY

SQ30/H/02

Mathematics
Paper 2

Date — Not applicable

Duration — 1 hour and 30 minutes

Total marks — 70

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

SQ30H02

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between **a** and **b**

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

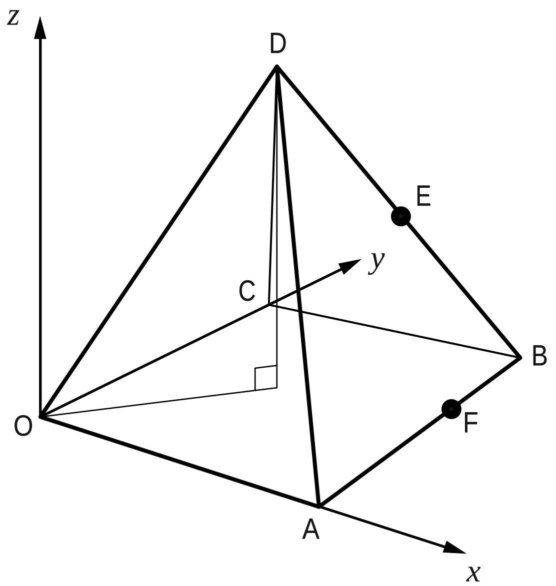
Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

Attempt ALL questions **MARKS**

Total marks — 70

1.



A square based right pyramid is shown in the diagram.

Square OABC has a side length of 60 units with edges OA and OC lying on the x-axis and y-axis respectively.

The coordinates of D are (30, 30, 80).

E is the midpoint of BD and F divides AB in the ratio 2:1.

(a) Find the coordinates of E and F. **2**

(b) Calculate $\overrightarrow{ED}.\overrightarrow{EF}$. **2**

(c) Hence, or otherwise, calculate the size of angle DEF. **4**

2. A wildlife reserve has introduced conservation measures to build up the population of an endangered mammal. Initially the reserve population of the mammal was 2000. By the end of the first year there were 2500 and by the end of the second year there were 2980.

It is believed that the population can be modelled by the recurrence relation:

$$u_{n+1} = au_n + b\ ,$$

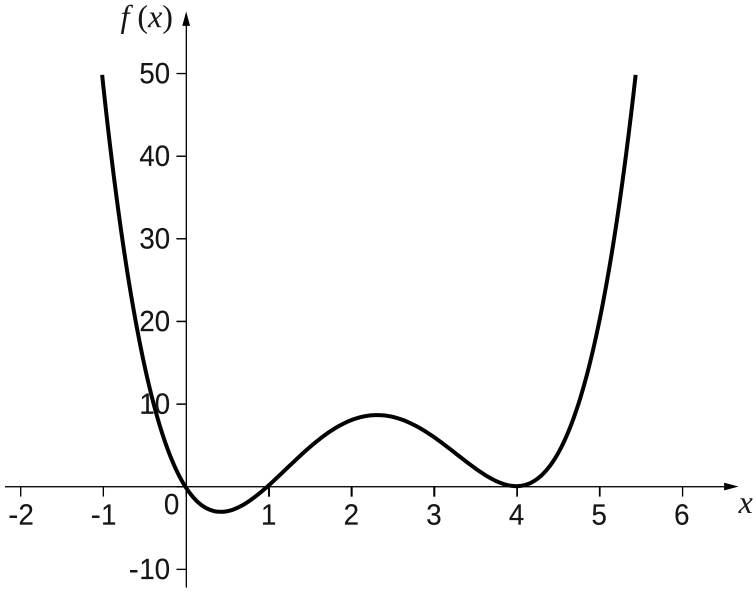
where a and b are constants and n is the number of years since the reserve was set up.

(a) Use the information above to find the values of a and b. **4**

(b) Conservation measures will end if the population stabilises at over 13 000. Will this happen? Justify your answer. **3**

MARKS

3. The diagram shows the graph of $f(x) = x(x-p)(x-q)^2$.



(a) Determine the values of p and q. **1**

(b) Find the equation of the tangent to the curve when $x = 1$. **4**

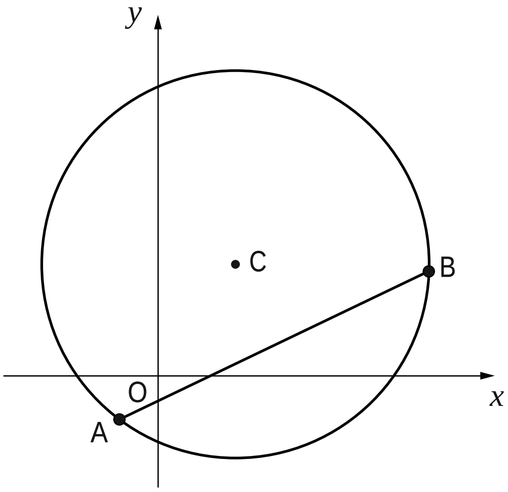
4. (a) Express $y = \log_4 2x$ in the form $y = \log_4 x + k$, clearly stating the value of k. **2**

(b) Hence, or otherwise, describe the relationship between the graphs of $y = \log_4 2x$ and $y = \log_4 x$. **1**

(c) Determine the coordinates of the point where the graph of $y = \log_4 2x$ intersects the x–axis. **2**

(d) Sketch and annotate the graph of $y = f^{-1}(x)$, where $f(x) = \log_4 2x$. **3**

5.



Points A(–1, –1) and B(7, 3) lie on the circumference of a circle with centre C, as shown in the diagram.

(a) Find the equation of the perpendicular bisector of AB. **4**

CB is parallel to the x-axis.

(b) Find the equation of the circle, passing through A and B, with centre C. **4**

6. The points A(0, 9, 7), B(5, –1, 2), C(4, 1, 3) and D(x, –2, 2) are such that AB is perpendicular to CD.

Determine the value of x. **5**

7. Given that $P(t) = 30e^{t-2}$ decide whether each of the statements below is true or false. Justify your answers.

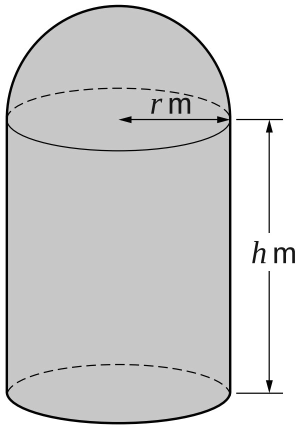
Statement A $\quad P(0) = 30$.

Statement B $\quad$ When $P(t) = 15$, the only possible value of t is 1·3 to one decimal place. **6**

MARKS

8. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.



(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$ **3**

(b) Determine the value of r which minimises the amount of metal needed to build the container. **6**

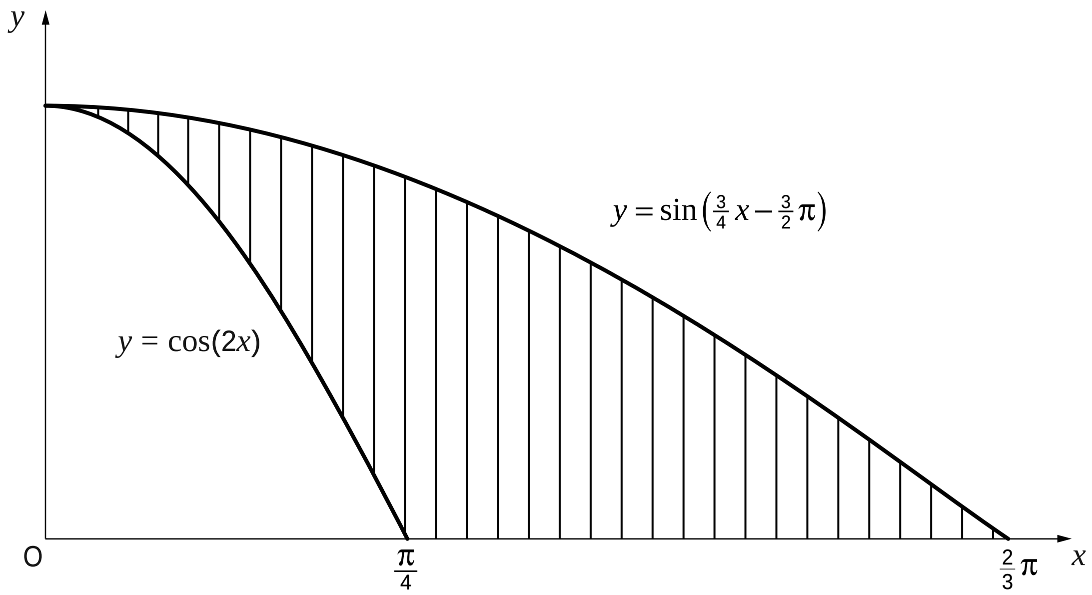
MARKS

9. A sea-life visitor attraction has a new logo in the shape of a shark fin.

The outline of the logo can be represented by parts of

- the x axis
- the curve with equation $y = \cos(2x)$
- the curve with equation $y = \sin\left(\frac{3}{4}x - \frac{3}{2}\pi\right)$

as shown in the diagram.

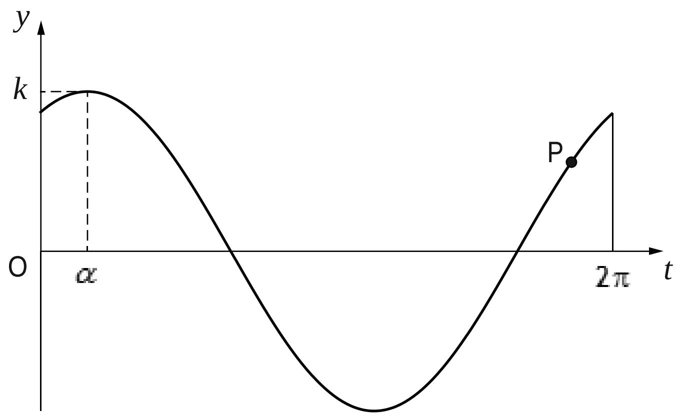


Calculate the shaded area. **6**

MARKS

10. Two sound sources produce the waves $y = \sin t$ and $y = \sqrt{3}\cos t$.

An investigation into the addition of these two waves produces the graph shown, with equation $y = k\cos(t - \alpha)$ for $0 \leq t \leq 2\pi$.



(a) Calculate the values of k and α. **4**

The point P has a y-coordinate of 1·2.

(c) Hence calculate the value of the t-coordinate of point P. **4**

[END OF SPECIMEN QUESTION PAPER]