## SQ30/H/01

# Mathematics Paper 1 <br> (Non-Calculator) 

Date —Not applicable
Duration —1 hour and 10 minutes

## Total marks - 60

Attempt ALL questions.

## You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.
State the units for your answer where appropriate.
Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.
Use blue or black ink.
Before leaving the examination room you must give your answer booklet to the Invigilator;
if you do not you may lose all the marks for this paper.

## FORMULAE LIST

## Circle:

The equation $x^{2}+y^{2}+2 g x+2 f y+c=0$ represents a circle centre $(-g,-f)$ and radius $\sqrt{g^{2}+f^{2}-c}$.
The equation $(x-a)^{2}+(y-b)^{2}=r^{2}$ represents a circle centre $(a, b)$ and radius $r$.

## Scalar Product:

$$
\text { or } \quad \mathbf{a} . \mathbf{b}=a_{1} b_{1}+a_{2} b_{2}+a_{3} b_{3} \text { where } \mathbf{a}=\left(\begin{array}{c}
a_{1} \\
a_{2} \\
a_{3}
\end{array}\right) \text { and } \mathbf{b}=\left(\begin{array}{l}
b_{1} \\
b_{2} \\
b_{3}
\end{array}\right)
$$

Trigonometric formulae:

$$
\begin{aligned}
\sin (\mathrm{A} \pm \mathrm{B}) & =\sin \mathrm{A} \cos \mathrm{~B} \pm \cos \mathrm{A} \sin \mathrm{~B} \\
\cos (\mathrm{~A} \pm \mathrm{B}) & =\cos \mathrm{A} \cos \mathrm{~B} \mp \sin \mathrm{~A} \sin \mathrm{~B} \\
\sin 2 \mathrm{~A} & =2 \sin \mathrm{~A} \cos \mathrm{~A} \\
\cos 2 \mathrm{~A} & =\cos ^{2} \mathrm{~A}-\sin ^{2} \mathrm{~A} \\
& =2 \cos ^{2} \mathrm{~A}-1 \\
& =1-2 \sin ^{2} \mathrm{~A}
\end{aligned}
$$

Table of standard derivatives:

| $f(x)$ | $f^{\prime}(x)$ |
| :---: | :---: |
| $\sin a x$ <br> $\cos a x$ | $a \cos a x$ <br> $-a \sin a x$ |

Table of standard integrals:

| $f(x)$ | $\int f(x) d x$ |
| :---: | :---: |
| $\sin a x$ | $-\frac{1}{a} \cos a x+C$ |
| $\cos a x$ | $\frac{1}{a} \sin a x+C$ |

## Total marks-60

1. Find $\int \frac{3 x^{3}+1}{2 x^{2}} d x, x \neq 0$.
2. Find the coordinates of the points of intersection of the curve $y=x^{3}-2 x^{2}+x+4$ and the line $y=4 x+4$.
3. In the diagram, $P$ has coordinates $(-6,3,9)$,
$\overrightarrow{P Q}=6 \mathbf{i}+12 \mathbf{j}-6 \mathbf{k}$ and $\overrightarrow{P Q}=2 \overrightarrow{Q R}=3 \overrightarrow{R S}$.

Find the coordinates of $S$.

5. Line $l_{1}$ has equation $\sqrt{3} y-x=0$.
(a) Line $l_{2}$ is perpendicular to $l_{1}$. Find the gradient of $l_{2}$.
(b) Calculate the angle $l_{2}$ makes with the positive direction of the $x$-axis.
6. (a) Find an equivalent expression for $\sin (x+60)^{\circ}$.
(b) Hence, or otherwise, determine the exact value of $\sin 105^{\circ}$.
7. (a) Show that $(x+1)$ is a factor of $x^{3}-13 x-12$.
(b) Factorise $x^{3}-13 x-12$ fully.
8. $f(x)$ and $g(x)$ are functions, defined on the set of real numbers, such that
$\mathrm{f}(x)=1-\frac{1}{2} x$ and $g(x)=8 x^{2}-3$.
(a) Given that $h(x)=g(f(x))$, show that $h(x)=2 x^{2}-8 x+5$.
(b) Express $h(x)$ in the form $a(x+p)^{2}+q$.
(c) Hence, or otherwise, state the coordinates of the turning point on the graph of $y=h(x)$.
(d) Sketch the graph of $y=h(x)+3$, showing clearly the coordinates of the turning point and the $y$-axis intercept.
9. (a) $A B$ is a line parallel to the line with equation $y+3 x=25$.

A has coordinates $(-1,10)$.
Find the equation of $A B$.
(b) $3 y=x+11$ is the perpendicular bisector of AB .

Determine the coordinates of B.
10. Find the rate of change of the function $f(x)=4 \sin ^{3} x$ when $x=\frac{5 \pi}{6}$.
11. The diagram shows the graph of $y=f^{\prime}(x)$. The $x$-axis is a tangent to this graph.

(a) Explain why the function $f(x)$ is never decreasing.
(b) On a graph of $y=f(x)$, the $y$-coordinate of the stationary point is negative. Sketch a possible graph for $y=f(x)$.
12. The voltage, $V(t)$, produced by a generator is described by the function $V(t)=120 \sin 100 \pi t, t>0$, where $t$ is the time in seconds.
(a) Determine the period of $V(t)$. 2
(b) Find the first three times for which $V(t)=-60$. 6

## [END OF SPECIMEN QUESTION PAPER]

